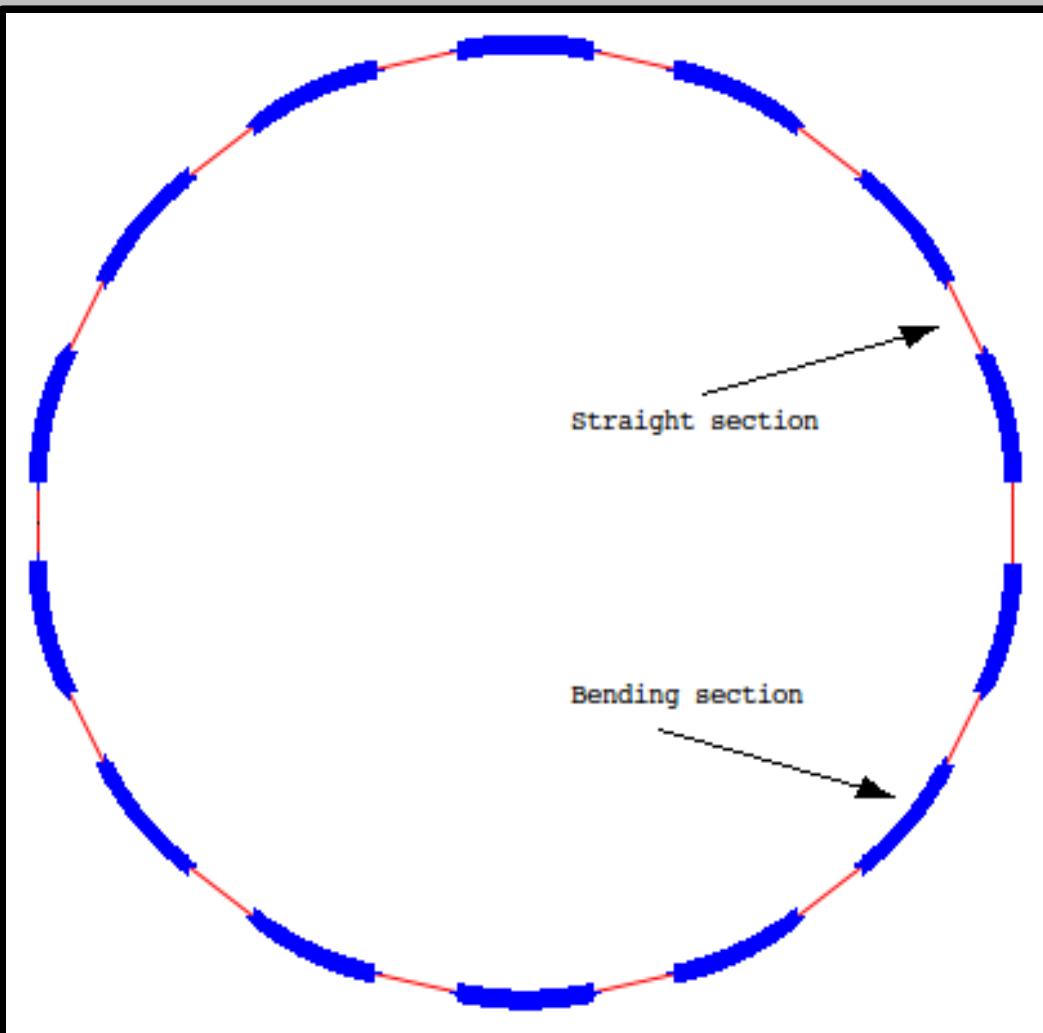


Spin Tracking - Method

$$\frac{d\vec{\beta}}{dt} = \frac{e}{\gamma mc} [\vec{E} - \vec{\beta}(\vec{\beta} \cdot \vec{E})]$$

$$\frac{d\vec{s}}{dt} = -\frac{e}{mc} \vec{s} \times \left[\left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right]$$

- 4th order Runge-Kutta
- $dt=0.5$ ps $\rightarrow 90\mu m$
- Spin and momentum on the same direction
- Too slow



Spin Tracking – Potential and E-field

$$V(r) = -\frac{E_0 R_0}{m} \left(\frac{R_0^m}{r^m} - 1 \right)$$

$$E_R = E_0 \left(\frac{R_0}{R} \right)^n \left[1 - \frac{n^2 - 1}{2} \frac{y^2}{R} + \frac{1}{24} (n^2 - 1)(n+1)(n+3) \frac{y^4}{R} + O(z^6) \right]$$

$$E_z = E_0 \left(\frac{R_0}{R} \right)^n \left[n - 1 \frac{y}{R} + \frac{1}{6} (n^2 - 1)(n+1) \frac{y^3}{R} + O(y^5) \right]$$

m=0.2 , n=-(m+1)

R0: Radius of the bending section

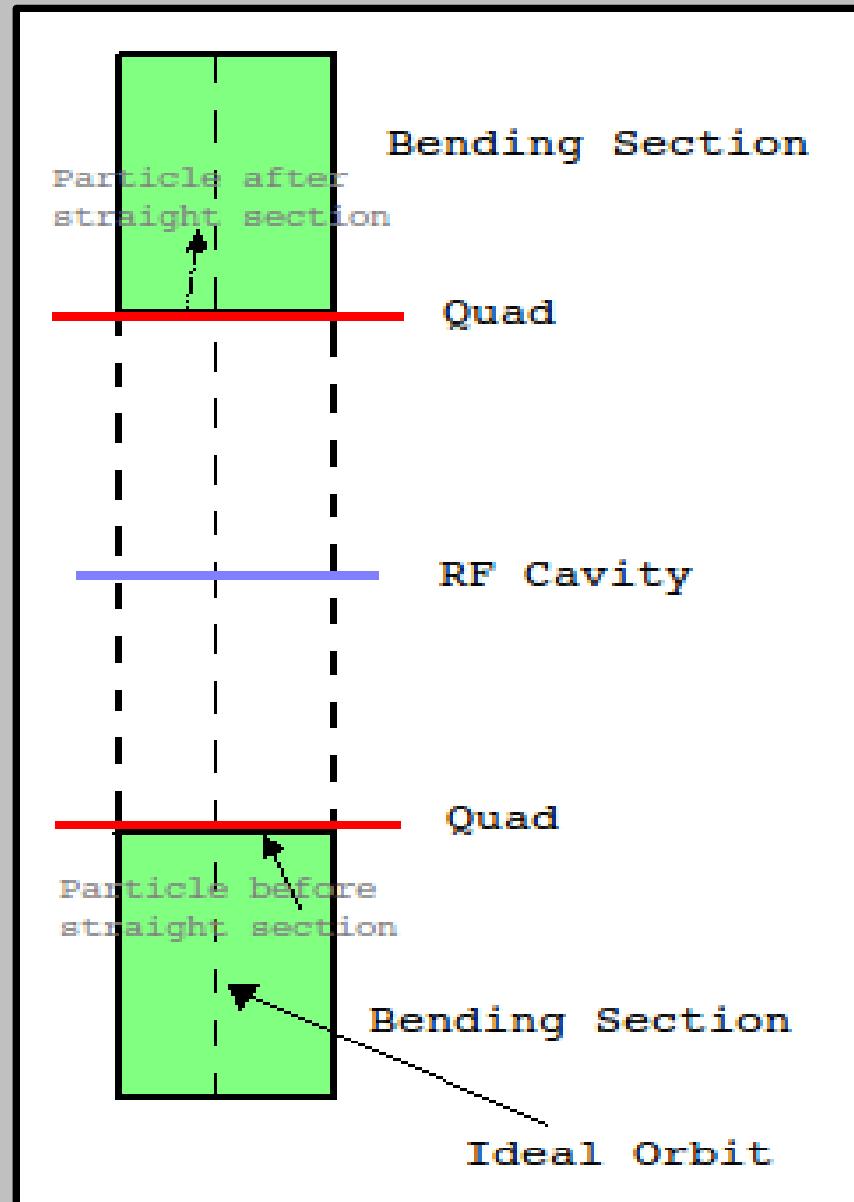
E0: E-field in the bending section

x,y: horizontal, vertical deviation of the particle

$$r = \sqrt{(R_0 + x)^2 + y^2} \quad R = R_0 + x$$

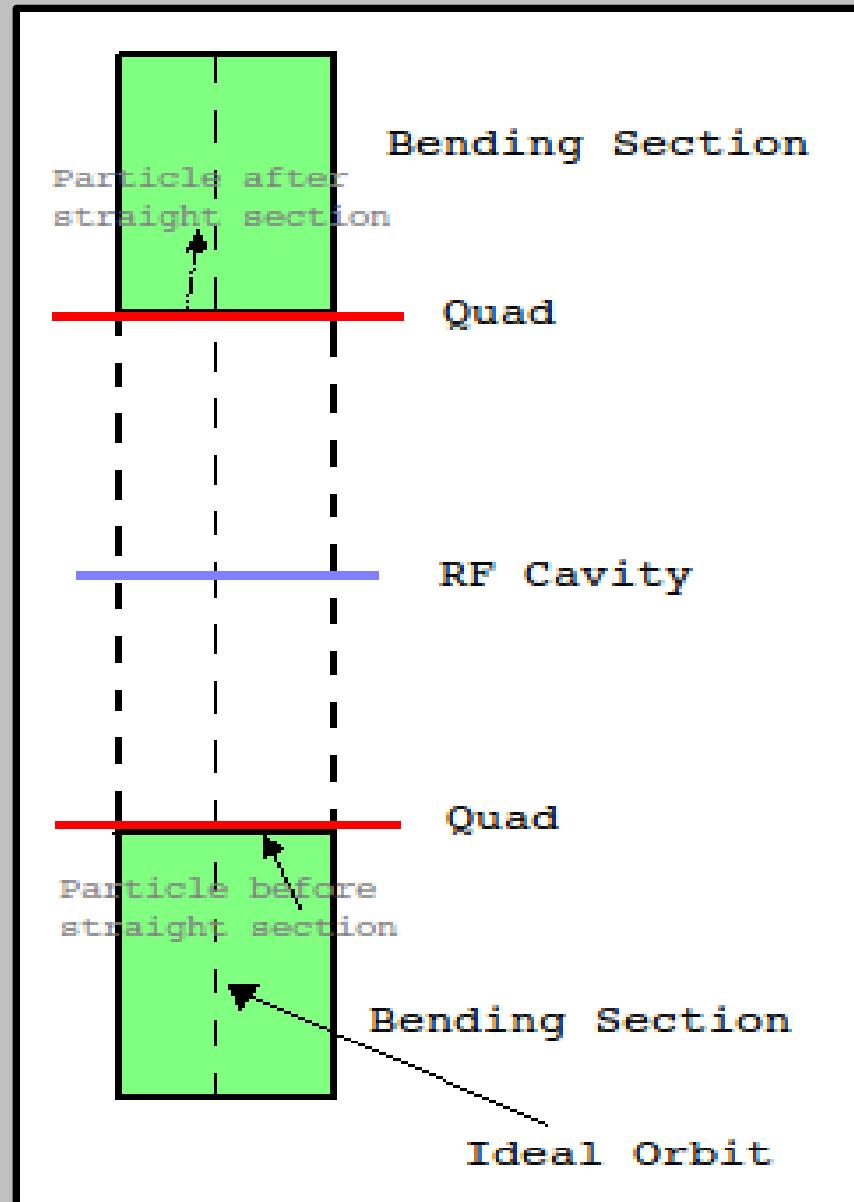
Spin Tracking – Straight Section

- Manual solution
- Fringe fields not taken into account
- Energy is conserved on boundaries
- Composed of:
 - Quads
 - RF Cavity
 - Empty section



Spin Tracking – RF Cavity

- In one straight section
- $E = eV_0 \cos(\omega t + \phi)$
- Spin is not updated



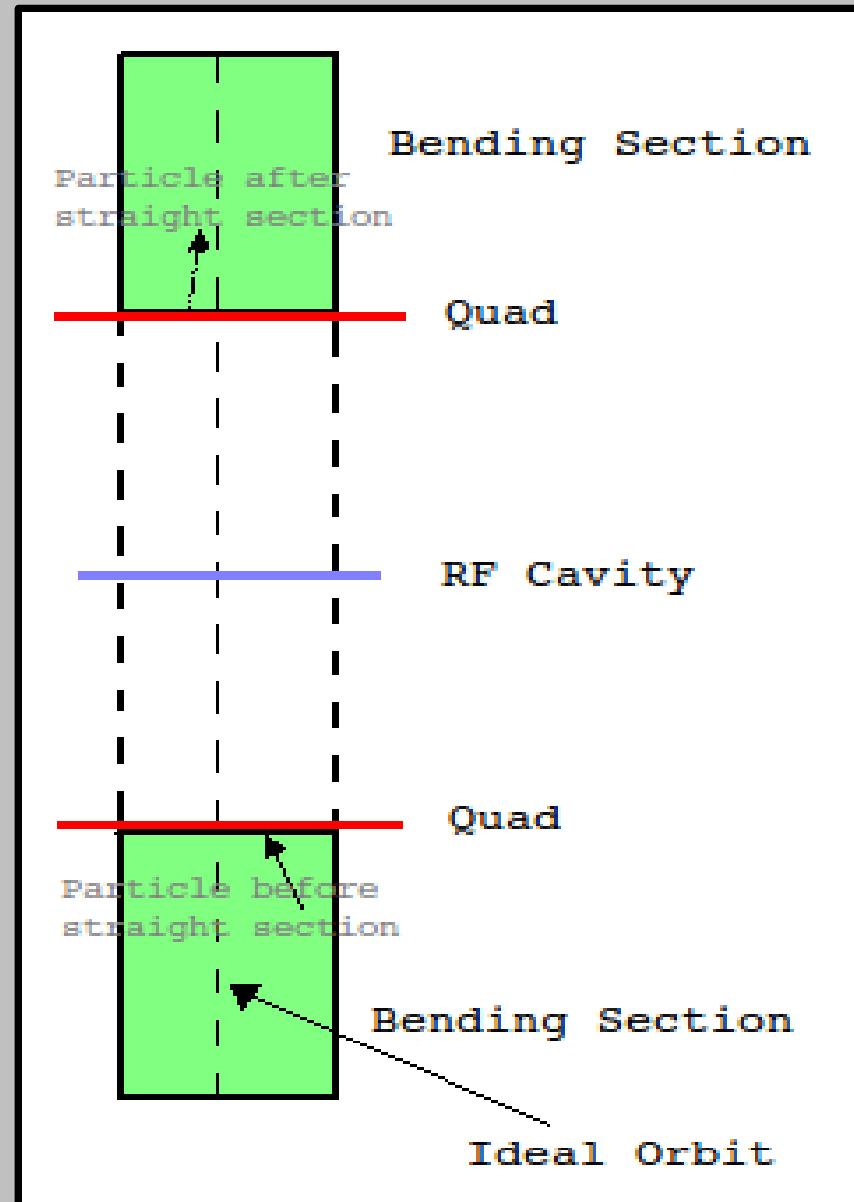
Spin Tracking – Quads

$$M_{F,D} = \begin{pmatrix} 1 & 0 \\ \mp kl & 1 \end{pmatrix}$$

$$k = \frac{q k_{quad}}{v^2 \gamma m}$$

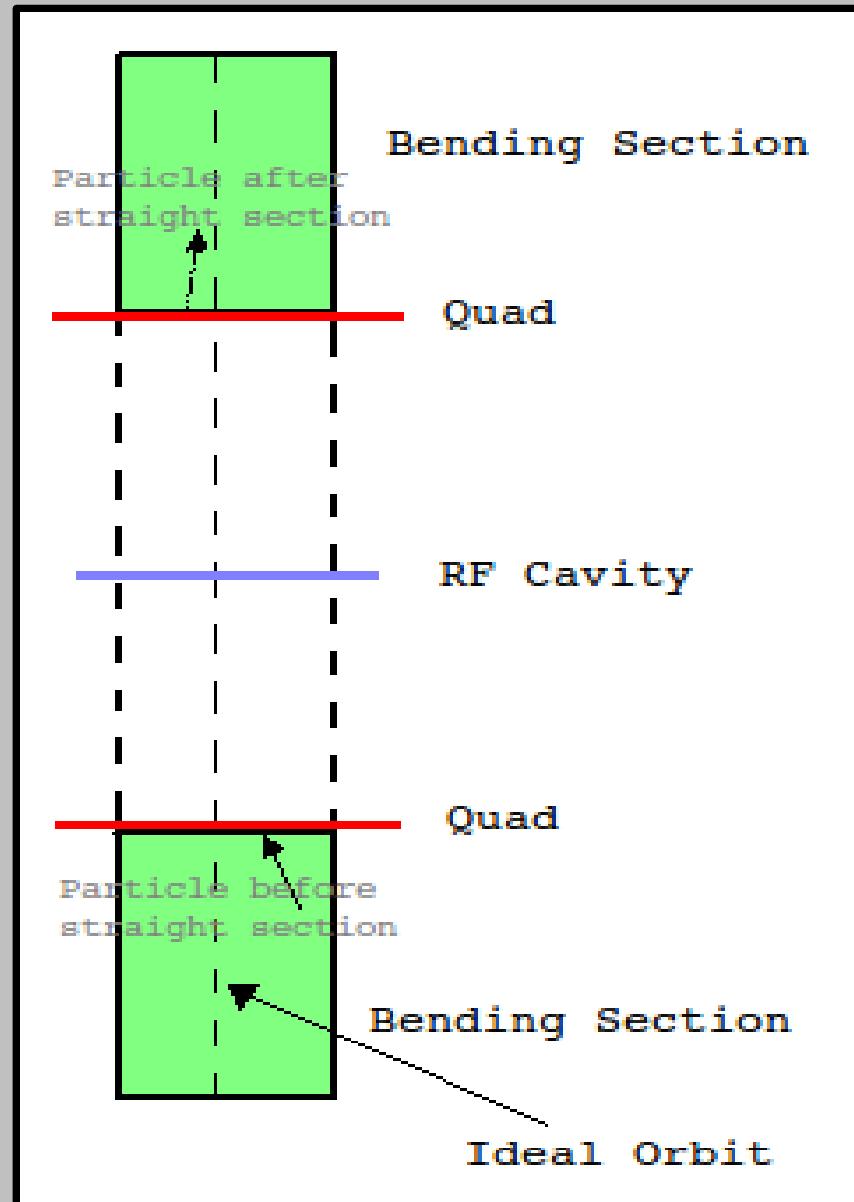
- Thin lens: $1e-10$ m
- Spin is updated as:

$$\Delta \vec{s} = \frac{d \vec{s}(\vec{\beta}, \vec{E}, \vec{s})}{dt} \Delta t$$



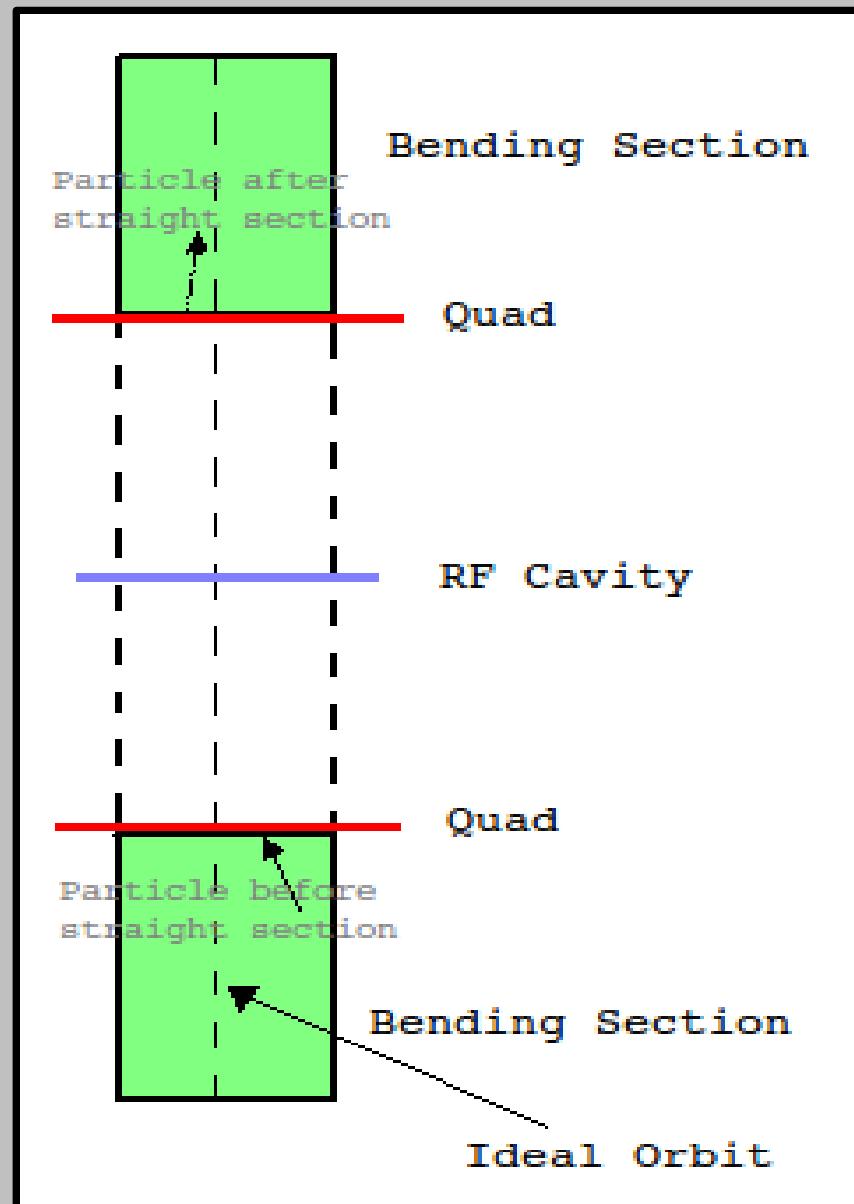
Spin Tracking – Empty Section

- Motion with no acceleration

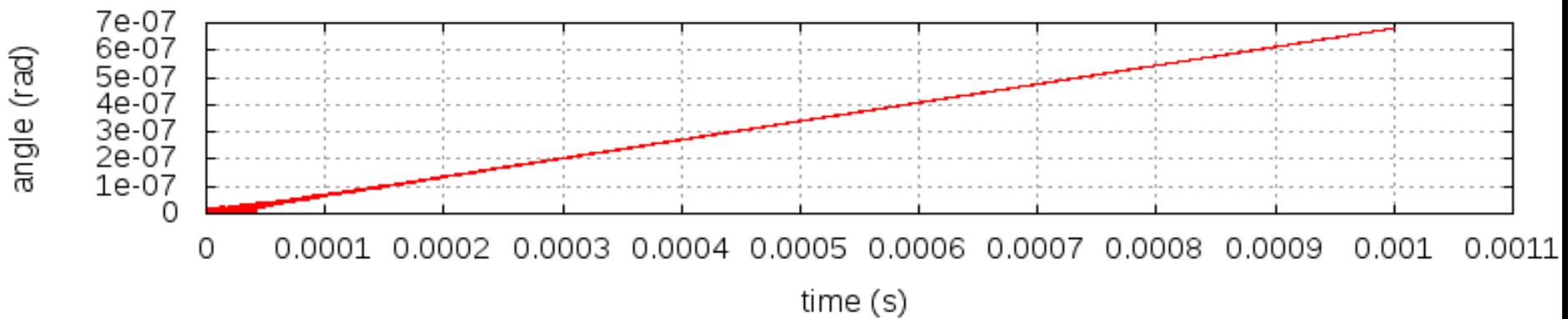
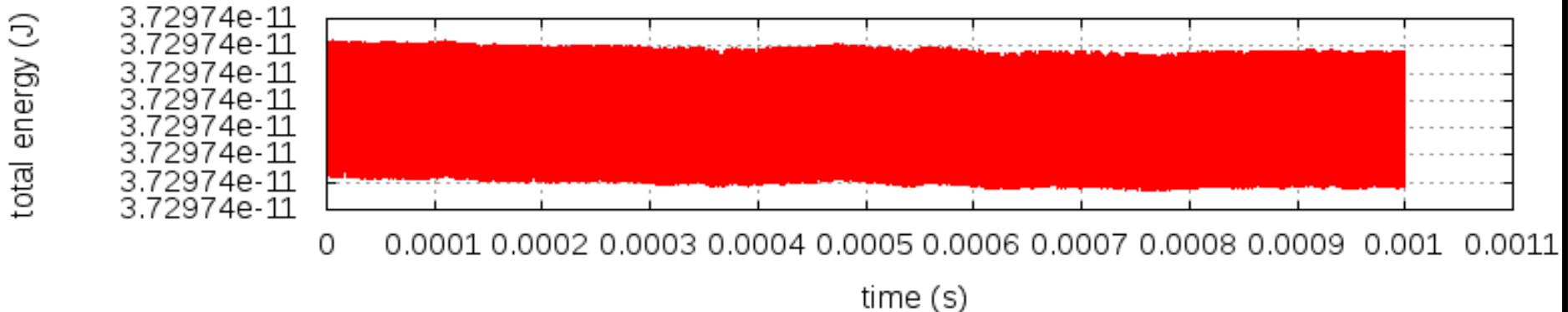


Spin Tracking – Boundary of Bending Section

- Different potential before and after straight section
- Longitudinal velocity is updated to conserve energy
- Spin is not updated

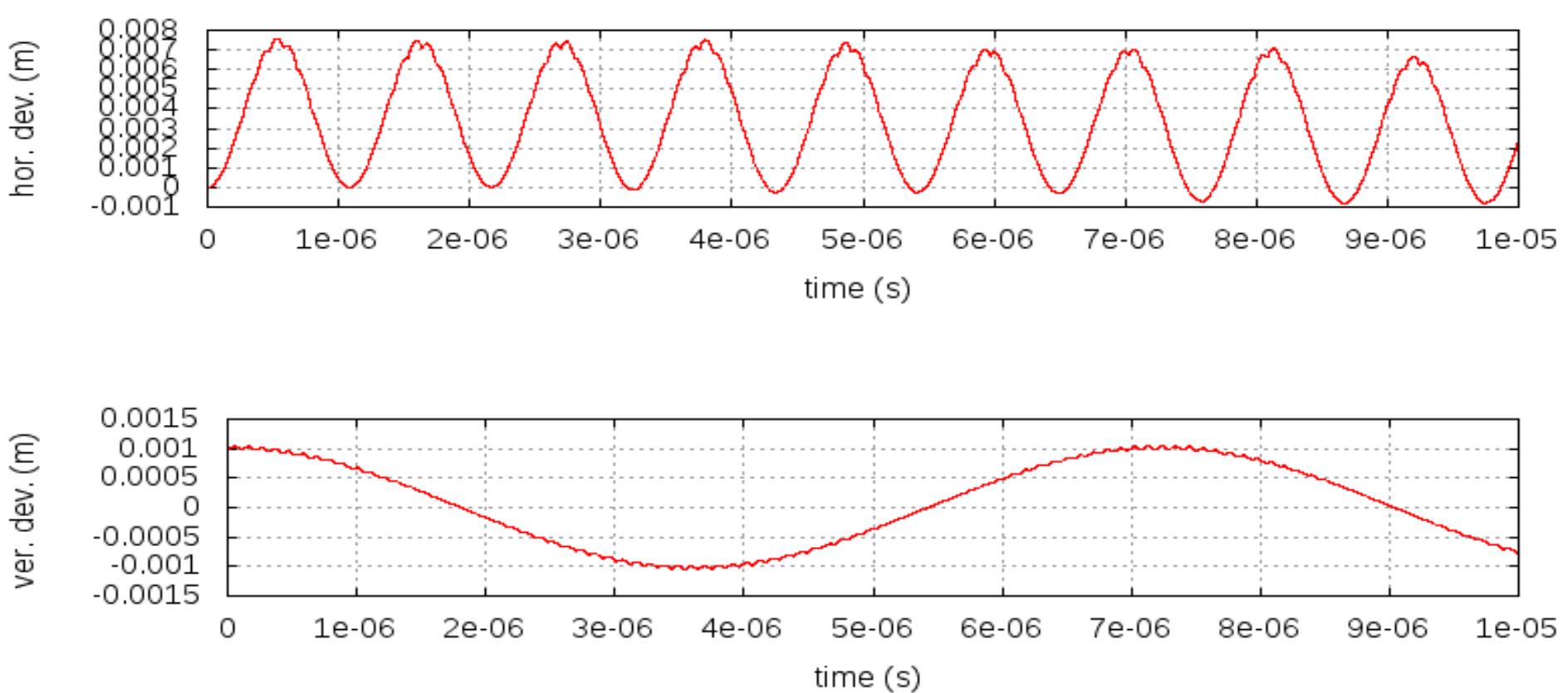


Spin Tracking – Limits of the method



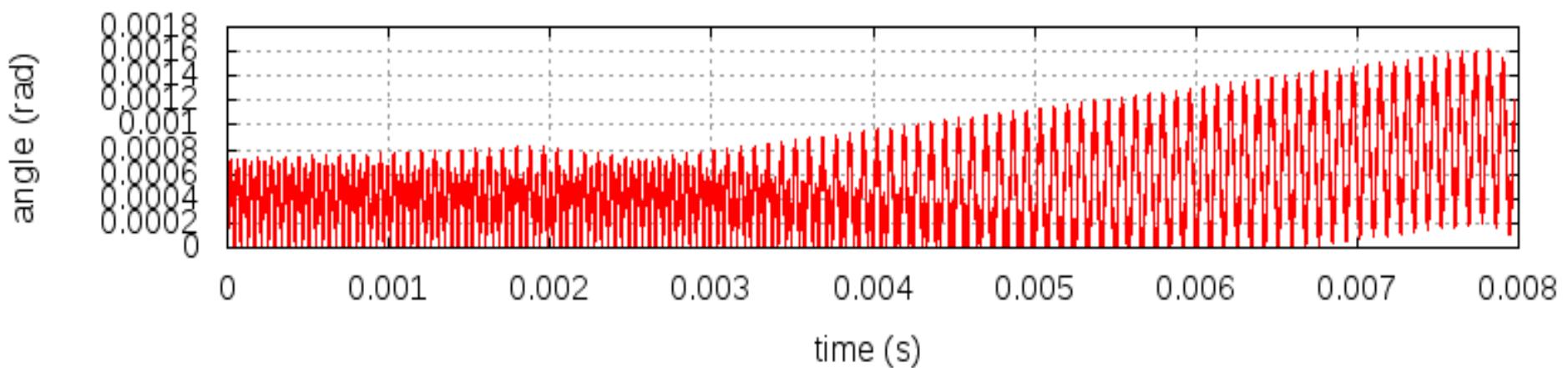
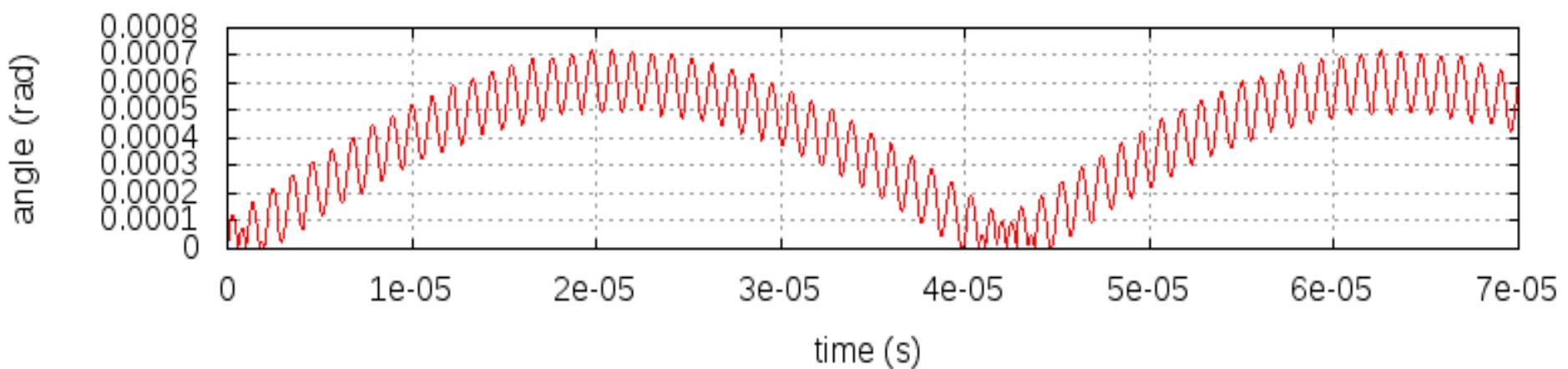
- No straight section, no quad, no rf cavity
- Energy is not conserved (Relative Error: 7.4e-12)
- Spin decoherence

Spin Tracking – Talman Lattice



- 14x1 m straight sections, $n=0.2$, rf on, quads on, off-magic momentum
- Horizontal tune: 1.36
- Vertical tune: 0.205

Spin Tracking – Talman Lattice – Spin Coherence



Spin Tracking - Plan

- ?